f. $\log_3(x) \cdot \log_9(x) = 2$

$$\log_3(x) \cdot \frac{\log_3(x)}{\underbrace{\log_3(9)}_{=2}} = 2$$

$$\frac{1}{2} \cdot (\log_3(x))^2 = 2$$

$$(\log_3(x))^2 = 4 \Rightarrow \begin{cases} \log_3(x) = 2 \Rightarrow \overbrace{3^{\log_3(x)}}^{=x} = 3^2 = 9 \\ \log_3(x) = -2 \Rightarrow \underbrace{3^{\log_3(x)}}_{=x} = 3^{-2} = \frac{1}{9} \end{cases}$$

Vérifions : $\underbrace{\log_3(9)}_{2} \cdot \underbrace{\log_9(9)}_{1} = 2$ ✓

$\underbrace{\log_3(\frac{1}{9})}_{-2} \cdot \underbrace{\log_9(\frac{1}{9})}_{-1} = 2$ ✓

$\underline{\underline{x_1 = 9}}, \; \underline{\underline{x_2 = \frac{1}{9}}}$

g. $\log(x^2) = \log^2(x)$

$2 \cdot \log(x) = \log^2(x)$ | Posons $y = \log(x)$

$2y = y^2$

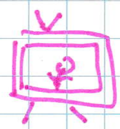
$y^2 - 2y = 0$

$y(y-2) = 0 \Rightarrow y = 0 \Rightarrow \log(x) = 0 \Rightarrow \underline{\underline{x_1 = 1}}$

$y = 2 \Rightarrow \log(x) = 2 \Rightarrow \underline{\underline{x_2 = 100}}$

<u>Vérifions :</u> $\log(1^2) = \log^2(1) \Rightarrow 0 = 0^2$ ✓

$\log(100^2) = \log^2(100) \Rightarrow 4 = 2^2$ ✓

<u>Exercice 6.14</u>  avec DESMOS

Observez les différences avec les courbes de $\ln(x)$ et e^x.

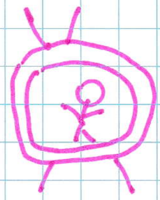
$\ln(x-1)$: décalage de 1 vers la droite ⚠ contre intuitif

$\ln(x+1)$: " " " " la gauche

$\ln(x)+1$: " " " vers le haut

$\ln(x)-1$: " " " vers le bas.

Exercice 6.15



$$N(t) = N_0\, e^{-\mu \cdot t}$$

a. $\cancel{5} \cdot \cancel{10^{10}} = \cancel{5} \cdot \cancel{10^{11}}\, e^{-1{,}2 \cdot 10^{-4} \cdot t}$

$$1 = 10 \cdot e^{-1{,}2\cdot 10^{-4} t}$$

$$\frac{1}{10} = e^{-1{,}2 \cdot 10^{-4} t} \quad | \ln(\,)$$

$$\ln(0{,}1) = \underbrace{\ln(e^{-1{,}2\cdot 10^{-4} t})}$$

$$\ln(0{,}1) = -1{,}2 \cdot 10^{-4}\, t$$

$$t = \frac{\ln(0{,}1)}{-1{,}2 \cdot 10^{-4}} = \frac{-2{,}3}{-1{,}2\cdot 10^{-4}} = 1{,}92 \cdot 10^4 = \underline{\underline{19'200 \text{ ans}}}$$

b. $\cancel{2{,}5 \cdot 10^{11}} = \cancel{5 \cdot 10^{11}}\, e^{-1{,}2 \cdot 10^{-4} t}$

$$\frac{1}{2} = e^{-1{,}2 \cdot 10^{-4} t} \quad | \ln(\,)$$

$$\ln(0{,}5) = \underbrace{\ln(e^{-1{,}2\cdot 10^{-4} t})}$$

$$\ln(0{,}5) = -1{,}2 \cdot 10^{-4} t$$

$$t = \frac{\ln(0{,}5)}{-1{,}2 \cdot 10^{-4}} = 0{,}5776 \cdot 10^4 = \underline{\underline{5776 \text{ ans}}}$$

Exercice 6.16

$$N(t) = N_0 \cdot 0{,}805^t$$

⚠ t est donné en millénaires

$t = 0{,}1 \Rightarrow 100$ ans $\Rightarrow 1$ siècle

nombre de mots qui restent après t millénaires.

a. $N(0{,}1) = N_0 \cdot 0{,}805^{0{,}1}$

$N(0{,}1) = 0{,}978\, N_0 \Rightarrow$ Il reste 97,8% des mots après 1 siècle

Donc 2,2% des mots ont disparu.

c. $\frac{1}{4} \cancel{N_0} = \cancel{N_0}\, 0{,}805^t$

il reste $\frac{1}{4}$ des mots

marche aussi avec ln()

$$\log(0{,}25) = \log(0{,}805^t) = t \cdot \log(0{,}805)$$

$$\Rightarrow t = \frac{\log(0{,}25)}{\log(0{,}805)} \simeq 6{,}4 \text{ millénaires, donc } \underline{\underline{6400 \text{ ans}}}.$$

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Exercice 6.17

$$P(h) = P_0 \cdot e^{-\alpha h}$$ ⚠ h en km.

a. $P(2) = 10^5 \cdot e^{-0,125 \cdot 2} = 0,7788 \cdot 10^5 = \underline{\underline{77880 \text{ Pa}}}$

b. $6 \cdot \cancel{10^4} = 10^{\cancel{5}} \cdot e^{-0,125 \cdot h}$

$$0,6 = e^{-0,125 h}$$

$$\ln(0,6) = \underbrace{\ln(e^{-0,125 h})}$$

$$-0,5108 = -0,125 h \quad \Rightarrow h \cong 4,1 \text{ km} \cong \underline{\underline{4000 \text{ mètres}}}$$

Exercice 6.18

a. $I = I_0 \cdot c^x$

$$\log(I) = \log(I_0 \cdot c^x)$$

$$\log(I) = \log(I_0) + \log(c^x)$$

$$\log(I) = \log(I_0) + x \cdot \log(c) \Rightarrow x = \underline{\underline{\frac{\log(I) - \log(I_0)}{\log(c)}}}$$

b. $x = \dfrac{\log(0,01 I_0) - \log(I_0)}{\log(0,25)}$

$$= \frac{\log\left(\frac{0,01 I_0}{I_0}\right)}{\log(0,25)} = \frac{\log(0,01)}{\log(0,25)} = \underline{\underline{3,32 \text{ m}}}$$

Exercice 6.19

$$h_1 = 12 \leftrightarrow \Pi_1 = 6,3 = \log\left(\frac{12}{y_0}\right) \Rightarrow 10^{6,3} = 10^{\log\left(\frac{12}{y_0}\right)} = \frac{12}{y_0}$$

$$h_2 = 3,6 \leftrightarrow \Pi_2 = \log\left(\frac{3,6}{y_0}\right) \qquad \Rightarrow y_0 = \frac{12}{10^{6,3}}$$

$$= \log\left(3,6 \cdot \frac{10^{6,3}}{12}\right) = \log(598578,6986) \cong \underline{\underline{5,8}}$$

$$h_3 = 1,1 \leftrightarrow \Pi_3 = \log\left(1,1 \cdot \frac{10^{6,3}}{12}\right) \cong \underline{\underline{5,3}}$$

$$h_4 = 1,0 \leftrightarrow \Pi_4 = \log\left(1,0 \cdot \frac{10^{6,3}}{12}\right) \cong \underline{\underline{5,2}}$$

Exercice 6.20

$10^{()}$ des 2 côtés

a. $\log(E) = 4{,}4 + 1{,}5 \cdot 7{,}2 = 15{,}2 \Rightarrow E = 10^{15,2} = \underline{\underline{1{,}585 \cdot 10^{15}}}$

b. $\log(E) = 4{,}4 + 1{,}5 \cdot 9{,}1 = 18{,}05 \Rightarrow E = 10^{18,05} = \underline{\underline{1{,}122 \cdot 10^{18}}}$

c. $\log(E) = 4{,}4 + 1{,}5 \cdot 4{,}2 = 10{,}7 \Rightarrow E = 10^{10,7} = \underline{\underline{5{,}012 \cdot 10^{10}}}$

d. $\log(E_7) = 4{,}4 + 1{,}5 \cdot 7 = 14{,}9 \Rightarrow E_7 = 10^{14,9}$

$\log(E_8) = 4{,}4 + 1{,}5 \cdot 8 = 16{,}4 \Rightarrow E_8 = 10^{16,4}$

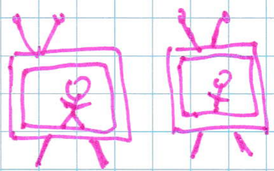
$$\Rightarrow \frac{E_8}{E_7} = \frac{10^{16,4}}{10^{14,9}} = 10^{16,4-14,9} = 10^{1,5} = \underline{\underline{31{,}62}}$$

e. $E = 15 \cdot 10^6 \cdot 4{,}2 \cdot 10^6 = 63 \cdot 10^{12} = 6{,}3 \cdot 10^{13}$

$$\log(6{,}3 \cdot 10^{13}) = 4{,}4 + 1{,}5 \cdot M$$

$$\log(6{,}3) + \underbrace{\log(10^{13})}_{=13} = 4{,}4 + 1{,}5\,M \Rightarrow M = \frac{\log(6{,}3) + 13 - 4{,}4}{1{,}5} = \underline{\underline{6{,}27}}$$

Exercice 6.21



$N(t) = N_0 e^{\beta \cdot t}$ $\qquad N_0$ et β inconnus.

Après 3 jours : $N(3) = 2 \cdot 10^5$ $\qquad$ Après 4,5 jours : $N(4{,}5) = 16 \cdot 10^5$

$2 \cdot 10^5 = N_0 \cdot e^{\beta \cdot 3} \Rightarrow N_0 = \dfrac{2 \cdot 10^5}{e^{3\beta}}$

$16 \cdot 10^5 = N_0 \cdot e^{\beta \cdot 4,5} \Rightarrow N_0 = \dfrac{16 \cdot 10^5}{e^{4,5\beta}}$

$$\frac{\cancel{2}^{1} \cdot \cancel{10^5}}{e^{3\beta}} = \frac{\cancel{16}^{8} \cdot \cancel{10^5}}{e^{4,5\beta}}$$

$$e^{4,5\beta} = 8 \cdot e^{3\beta}$$

$$\frac{e^{4,5\beta}}{e^{3\beta}} = \boxed{8 = e^{1,5\beta}} \Rightarrow \ln(8) = \underbrace{\ln(e^{1,5\beta})}_{1,5\beta} \Rightarrow \beta = \frac{\ln(8)}{1{,}5} \cong \underline{\underline{1{,}3863}}$$

$$\Rightarrow N_0 = \frac{2 \cdot 10^5}{e^{3 \cdot 1,3863}} = \frac{2 \cdot 10^5}{64} = \underline{\underline{3125}}$$

(15)

a. $N(5) = 3125 \cdot e^{1,3863 \cdot 5} = 32 \cdot 10^5 = 3,2 \cdot 10^6$

b. $8 \cdot 10^5 = 3125 \cdot e^{1,3863 \cdot t}$

$$\frac{8 \cdot 10^5}{3125} = e^{1,3863 t}$$

$$256 = e^{1,3863 t}$$

$$\ln(256) = 1,3863 t \Rightarrow \underline{\underline{t = 4}}$$

Exercice 6.21 c